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Reliability of structures by using probability and fatigue theories

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Abstract

Methodologies to calculate failure probability and to estimate the reliability of fatigue loaded structures are developed. The applicability of the methodologies is evaluated with the help of the fatigue crack growth models suggested by Paris and Walker. The probability theories such as the FORM (first order reliability method), the SORM (second order reliability method) and the MCS (Monte Carlo simulation) are utilized. It is found that the failure probability decreases with the increase of the design fatigue life and the applied minimum stress, the decrease of the initial edge crack size, the applied maximum stress and the slope of Paris equation. Furthermore, according to the sensitivity analysis of random variables, the slope of Pairs equation affects the failure probability dominantly among other random variables in the Paris and the Walker models.

Keywords: Fatigue; Reliability; Failure Probability; Sensitivity; FORM; SORM; Monte Carlo simulation

1. Introduction

The repeated loads to structures may lead to failure of material even if the load level is much lower than the ultimate states. Many mechanical structures such as train axles and wheels, load-bearing parts of automobiles, offshore structures, and bridges are designed to endure for a long term up to giga-cycle loadings in the actual service. Furthermore, some mechanical structures in various areas need to be investigated if the operation life can be extended beyond the design life because of the economic consideration. In such circumstances, mechanical components of these structures are generally exposed to tremendous number of stress/strain cycles in the long service period. Thus, the fatigue properties of the structural materials under long service cyclic loadings are important subjects requiring the technical information for safety design of such mechanical structures [1-3].

In the fatigue design, the use of S-N curves has been well established. These curves predict fatigue failure under constant amplitude loading, but cannot incorporate information related to crack initiation and/or propagation. If a crack is discovered in a mechanical structure during an operation, the structures can be repaired for a further use. The popular S-N approach is no longer useful for this circumstance. However, the fracture mechanics techniques can be successfully applied to this problem. The fracture mechanics need the information about the defects and/or cracks for such analysis. Since the size and location of defects are quite random, a deterministic analysis may provide incomplete results about the structure reliability. Also, the statistical random character of loads, geometry and material properties may affect significantly the reliability of a structure. Therefore, the fracture mechanics with the help of the probabilistic method provide a useful tool to solve these problems [4-7].

In this paper, fatigue models suggested by Paris and Walker are used to formulate the limit state func-

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tion (LSF) for assessing the failure of fatigue damaged structures. And the failure probability is estimated by using the FORM (first order reliability method) and the SORM (second order reliability method). The reliabilities of the fatigued structures are assessed by using this failure probability. The applicability of these methods to the reliability estimation is estimated by providing a case study. Furthermore, the sensitivity of each random variable to the reliability, which is quantifying the effect on the failure probability, is estimated.

The results obtained by using the FORM and the SORM are compared with those estimated by using the MCS (Monte Carlo simulation) to check the accuracy of the proposed methodologies in this paper.

2. Fatigue models

The strength of a component or structure can be significantly reduced by the presence of a crack. The fatigue crack growth rate, da/dN, versus the applied stress intensity factor range, ΔK, can be obtained from fatigue crack propagation experiments. The corresponding applied stress intensity factor range, ΔK, is calculated when the crack length, a, and the applied stress range, ΔS, are measured in the experiments as below [1,3,4].

$$\begin{aligned} \Delta K_I &= \Delta K = K_{\max} - K_{\min} \\ &= S_{\max}\sqrt{\pi a}\alpha - S_{\min}\sqrt{\pi a}\alpha \\ &= (S_{\max} - S_{\min})\sqrt{\pi a}\alpha = \Delta S\sqrt{\pi a}\alpha \end{aligned} \quad (1)$$

Where, α is a geometric parameter, $K_{\max}$ is the maximum stress intensity factor, $K_{\min}$ is the minimum stress intensity factor, $S_{\max}$ is the maximum applied stress and $S_{\min}$ is the minimum applied stress.

Since the stress intensity factor is undefined in the compression, $K_{\min}$ is taken as zero, if $S_{\min}$ is compressive. The correlation for constant amplitude loading is usually plotted on a log-log sheet with the fatigue crack growth rate, da/dN, in $m/cycle$, versus the opening mode stress intensity factor range, ΔK_I (or ΔK), in $MPa\sqrt{m}$.

A typical log-log plot of fatigue crack growth rate versus stress intensity factor range, as shown schematically in Fig. 1, has a sigmoid shape that can be divided into three major regions. Region I is the near

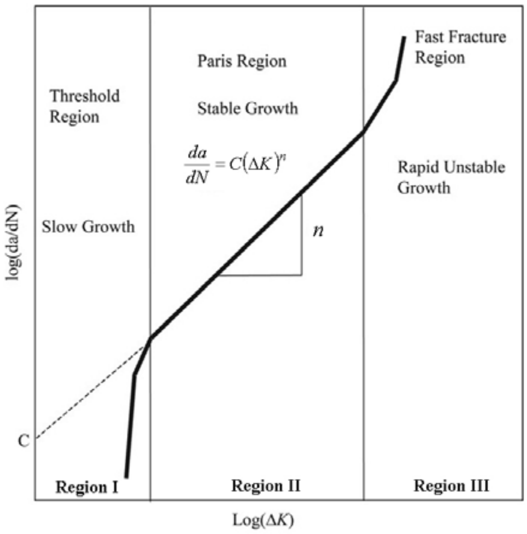


Fig. 1. Schematic behavior of fatigue crack growth rate versus stress intensity factor range.

threshold region that indicates a threshold value, ΔK_{th}, and there is no observable crack growth below this value. This threshold occurs at crack growth rates on the order of $1\times10^{-10} m/cycle$ or less. Region II shows essentially a linear relationship between log da/dN and $\log\Delta K$, which corresponds to the formula suggested by Paris [1,3,4].

$$\frac{da}{dN} = C(\Delta K)^n \quad (2)$$

Where, n, C are material constants. n is the slope of the straight portion and C is the coefficient found by extending the straight line to $\Delta K = 1MPa\sqrt{m}$. In region II, the fatigue crack growth corresponds to stable macroscopic crack growth. This is typically controlled by the boundary conditions. The microstructure and mean stress show less influence on fatigue crack growth behavior in region II than in region I. In region III, the fatigue crack growth rates are very high as it approaches instability, and little fatigue crack growth life is involved. This region is controlled primarily by fracture toughness K_C or K_{IC}, which depends on the microstructure, mean stress, and environment.

Conventional S-N or ε-N fatigue behavior is usually referenced to the fully reversed stress or strain conditions ($R=-1$). However, fatigue crack growth data are usually referenced to the pulsating tension condition with $R=0$ or approximately zero.

The general influence of mean stress on fatigue crack growth behavior can be estimated by using the stress ratio, $R = K_{\min} / K_{\max} = S_{\min} / S_{\max}$, which is used as the principal parameter and has a positive value, $R \geq 0$. It should be recognized that the effect of the R ratio on the fatigue crack growth behavior is strongly material dependent.

A common empirical relationship used to include mean stress effects with $R \geq 0$ is the Walker equation shown as below [1,3-5].

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1-R)^{n(1-\lambda)}} = C"(\Delta K)^n \tag{3}$$

Where, C and n are the coefficient and slope of Paris equation for $R = 0$, respectively, and λ is a material constant. The Paris equation and the Walker equation are basically similar, with different coefficients of the equations, C and $C"$, as below.

$$C" = \frac{C}{(1-R)^{n(1-\lambda)}} \tag{4}$$

Because the effect of R on fatigue crack growth is known as material dependent, it is necessary to determine the material constant, λ. The value of λ for various metals ranges from 0.3 to nearly 1, with a typical value of around 0.5.

The fatigue failure life, N_{ff}, can be obtained by integrating the fatigue crack growth rate formula at the domain from initial crack, a_i, to final crack, a_f. And the final crack can be calculated by using the fracture toughness as below.

$$a_f = \frac{1}{\pi}\left(\frac{K_c}{S_{\max}\alpha}\right)^2 \tag{5}$$

3. Failure probability

3.1 FORM (first order reliability method)

The failure probability is calculated by using the FORM, which is one of the methods utilizing the reliability index. The FORM method is based on the first-order Taylor series approximation of an LSF, which is defined as below [6-10].

$$Z = RE - LO \tag{6}$$

Where, RE is the resistance normal variable, and LO is the load normal variable. Assuming that RE and LO are statistically independent, normally distributed random variables, the variable Z is also normally distributed. The failure occurs when $RE < LO$, i.e., $Z < 0$. The failure probability is given as below.

$$\begin{aligned} PF &= P[Z<0] \\ &= \int_{-\infty}^{0} \frac{1}{\sigma_Z\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{Z-\mu_Z}{\sigma_Z}\right)^2\right\} dZ \\ &= \int_{-\infty}^{-\beta} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{U^2}{2}\right\} dU = \Phi(-\beta) \end{aligned} \tag{7}$$

Where, μ_Z and σ_Z are the mean and standard deviation of the variable Z, respectively, and Φ is the cumulative distribution function for a standard normal variable, and β is the safety index or reliability index and the coefficient of variance (C.O.V.) denoted as below.

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_L}{\sqrt{\sigma_R^2 + \sigma_L^2}} \tag{8}$$

$$C.O.V. = \frac{\sigma_{X_i}}{\mu_{X_i}} \tag{9}$$

Where, μ_{X_i} and σ_{X_i} are the mean and standard deviation of the variable X_i, respectively

Eq. (8) can be used when the system has a linear LSF. Actually, most real systems and cases do not have linear LSF but rather have a nonlinear LSF. So, for a system that has a nonlinear LSF, Eq. (8) cannot be used to calculate the reliability index. Rackwitz and Fiessler proposed a method to estimate the reliability index that uses the procedure shown in Fig. 2 for a system having a nonlinear LSF. In this paper, we iterate the loop, as shown in Fig. 2, to determine a reliable reliability index until the reliability index converges to a desired value ($\Delta\beta \leq 0.001$) [9, 10].

The LSF must be defined to formulate the FORM and evaluate the reliability. In this paper, the LSF can be defined by using the fatigue models as below [6,7].

$$Z = N_D - N_{ff} \tag{10}$$

Where, N_D is the design fatigue life that is a required value in the design procedure of structure and N_{ff} is the fatigue failure life estimated from the fatigue crack growth models such as the Paris and the Walker models by using Eq. (2) or Eq. (3).

The sensitivity index, which is used to evaluate the effect of random variables on the failure probability, is denoted as below [9, 10].

$$SI_{X_i} = \frac{\left(\frac{\partial Z}{\partial X_i}\right)}{\left(\sqrt{\sum\left(\frac{\partial Z}{\partial X_i}\right)^2}\right)} \quad (11)$$

Where SI_{X_i} is the sensitivity index of a random variable X_i and $\partial Z/\partial X_i$ is the partial derivative of a random variable X_i.

3.2 SORM (second order reliability method)

The computations required for reliability analysis of systems with linear LSF are relatively simple. However, the LSF could be nonlinear either due to a nonlinear relationship of the random variables in the LSF or due to some variables being non-normal.

The FORM approach will give the same reliability index for both linear and nonlinear limit state cases, if the minimum distance point is the same. But it is apparent that the failure probability of the nonlinear limit state would be less than that of the linear limit state, due to the difference in the failure domains. The curvature of the limit state around the minimum distance point determines the accuracy of the first order approximation in the FORM. The SORM improves the FORM result by including additional information about the curvature of the limit state.

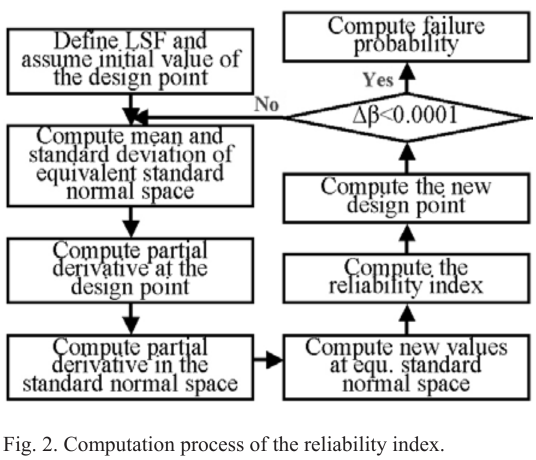


Fig. 2. Computation process of the reliability index.

Fiessler first explored the SORM approach by using various quadratic approximations. A simple closed form solution for probability computation by using a second order approximation and with a help of the theory of the asymptotic approximation was given by Breitung [6,7,9].

$$PF_{SORM} = \Phi(-\beta)\prod_{i=1}^{n-1}(1-\beta\kappa_i)^{-1/2} \quad (12)$$

Where κ_i denotes the principal curvatures of the LSF at the minimum distance point and β is the reliability index calculated by using the FORM. The principal curvatures are computed by using steps shown in Fig. 3.

3.3 MCS (Monte Carlo Simulation)

Unlike many engineering analytical results, the ones obtained by probabilistic methods are difficult to verify experimentally. Instead, we use the MCS technique to verify the accuracy of the results obtained from the FORM. Most MCSs are usually performed in engineering applications by the steps shown in Fig. 4 [6, 7, 9, 10].

In the MCS, many simulations are performed. In each simulation, the values of the variables are randomly generated according to their probability density functions. And then the LSF is used to evaluate the performance function in each simulation. Finally, the probability $P[Z<0]$ is estimated as:

$$P[Z<0] = \frac{N_f}{N} \quad (13)$$

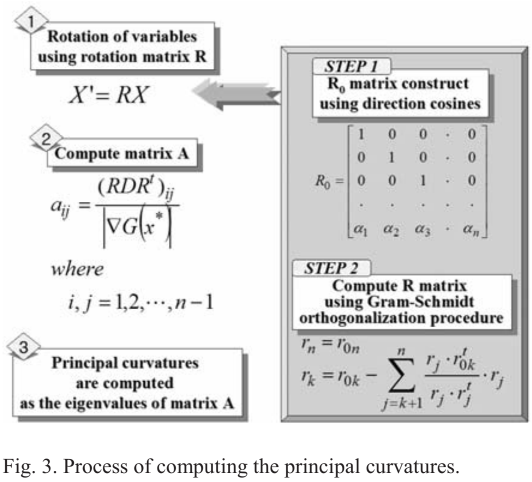


Fig. 3. Process of computing the principal curvatures.

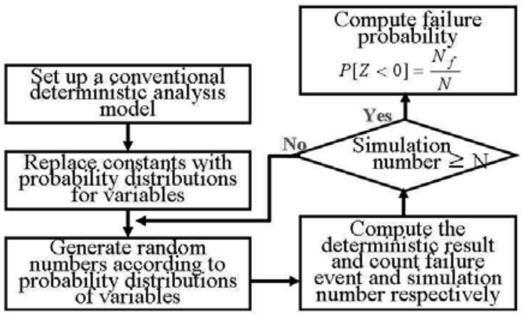


Fig. 4. Computation process of the failure probability by the Monte Carlo simulation.

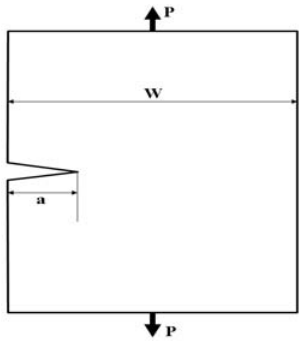


Fig. 5. The geometry of single edge crack specimen.

Where, N_f is the number of simulations with $Z<0$, which is number of failures, and N is the total number of simulations. To obtain reliable results, the total number of simulations, N, is selected as:

$$N \geq \frac{100}{PF} \tag{14}$$

In this paper, N is chosen as 10^8 because we set the maximum target safety level of the buried pipeline corresponding to $PF=10^{-6}$ to ensure the perfect integrity of the buried pipeline.

4. A case study

In this paper, we formulate the LSF by using fatigue models, and estimate the failure probability by using the FORM and the SORM with the given data for fatigue experiment data in a single edge crack shown in Fig. 5. The specimen is a very wide SAE 1020 cold-rolled thin plate subjected to constant amplitude uni-axial cyclic loads. The random variables and their statistical values used in the fatigue models are listed in Table 1 [1-5].

Table 1. Random variables and its statistical values used in a case study.

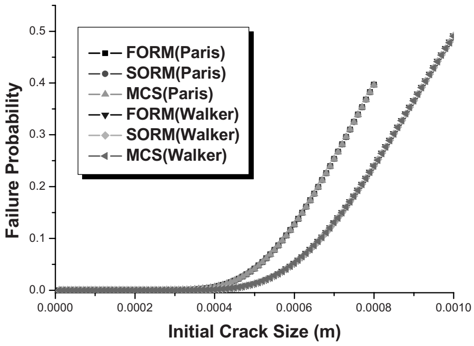
Valuable	Mean	C.O.V
S_{max}	200 MPa (Paris)	0.002
	300 MPa (Walker)	0.002
S_{min}	-50 MPa (Paris)	0.002
	100 MPa (Walker)	0.002
S_y	630 MPa	-
S_u	670 MPa	-
E	207 GPa	-
K_c	104 $MPa\sqrt{m}$	-
a_i	0.001 m	0.01
α	1.12	-
C	6.9×10^{-12}	0.02
n	3.0	0.02
N_D	129,000 $cycle$ (Paris)	0.003
	65000 $cycle$ (Walker)	0.003
λ	0.5	-

5. Results and discussions

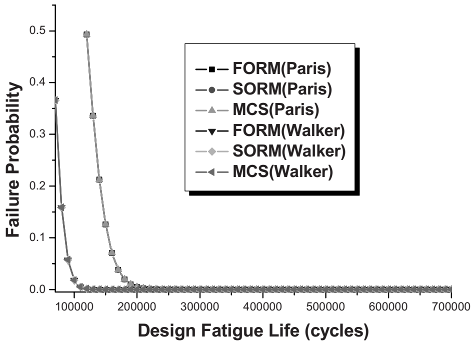
In this paper, the LSF is formulated by using the fatigue crack growth models suggested by Paris and Walker. The failure probability is estimated by using the values of random variables listed in Table 1 and probability theories such as the FORM, the SORM and the MCS.

The relationship between failure probability and variation of random variables is shown in Fig. 6, corresponding to the fatigue models and the probability theories. It is found from Fig. 6 that the failure probability decreases with the increase of the design fatigue life and the applied minimum stress, the decrease of the initial edge crack size, the applied maximum stress and the slope of Paris equation. The specific statistical values are used in a deterministic case study to compare the results out of the Pairs model to the Walker model. It is found in Fig. 6 that the failure probabilities based on the Paris and Walker models turn out to be very similar for variation of the initial edge crack size and the slope of the Paris equation.

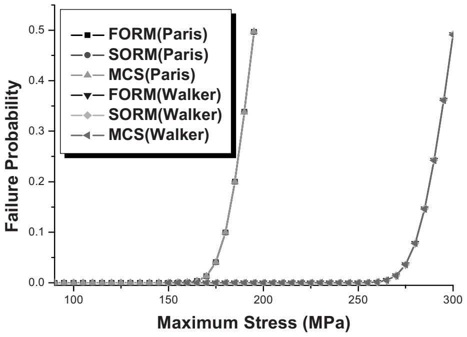
However, the Paris and Walker models show different failure probabilities with variation of the design fatigue life, because they have different fatigue lives corresponding to the maximum and minimum stresses.



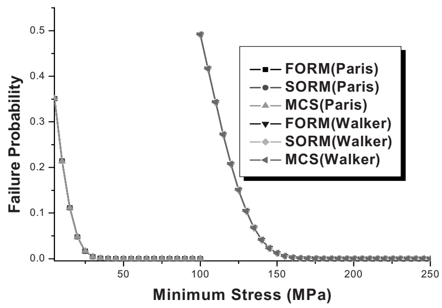
(a) Initial edge crack size



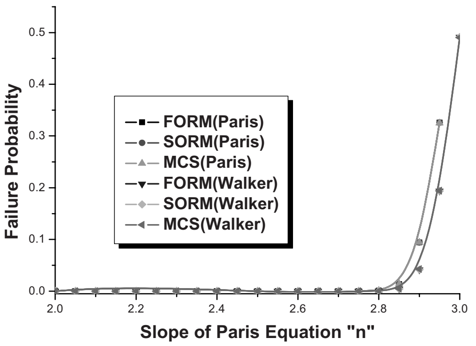
(b) Design fatigue life



(c) Applied maximum stress



(d) Applied minimum stress



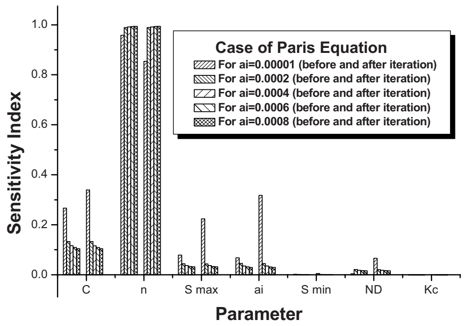
(e) Slope of Paris equation

Fig. 6. Relationship between failure probability and various random variables according to the FORM, the SORM and the MCS.

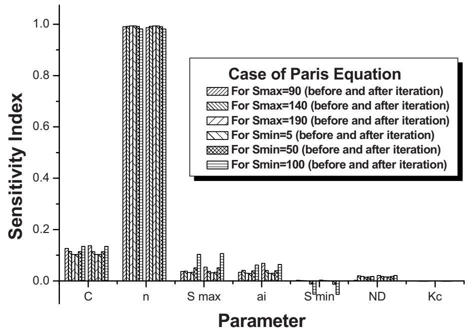
It is found from Fig. 6 that the FORM, the SORM and the MCs show similar failure probability for the Paris and Walker models. Table 2 quantitatively shows the mean percentile differences among the results of the FORM, the SORM and the MCS for the Paris and Walker models, respectively. For the Paris and the Walker models from Table 2 the FORM and the SORM show a similar failure probability for varying random variables. On the other hand, the difference in failure probability between the FORM and the MCS is similar to that between the SORM and the MCS.

It is also recognized from Table 2 that the Walker model shows slightly larger differences in failure probability among the FORM, the SORM and the MCS than the Paris model for the variation of the initial edge crack size, the applied minimum stress and the slope of Paris equation.

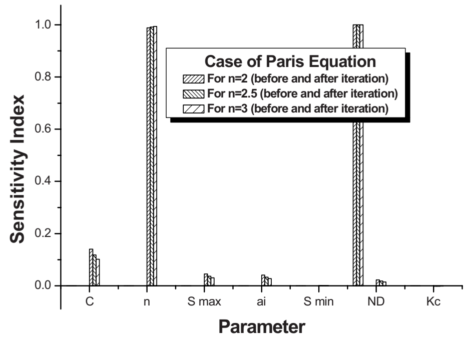
On the other hand, the Paris model shows slightly larger differences in failure probability among the FORM, the SORM and the MCS than the Walker model for the variation of the applied maximum stress



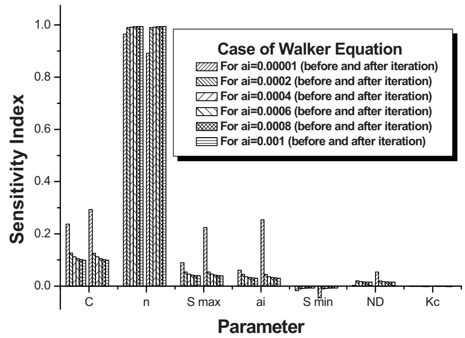
(a) Change in initial edge crack



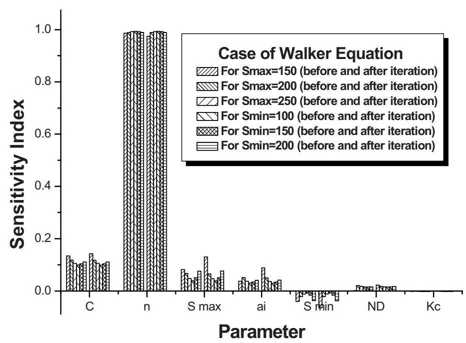
(b) Change in applied maximum and minimum stresses



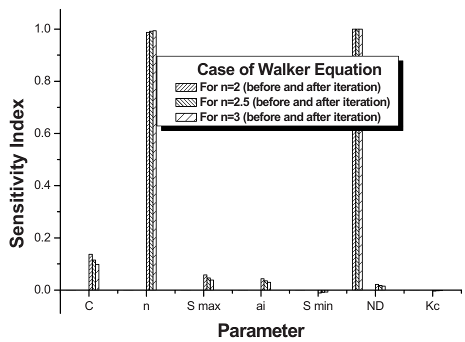
(c) Change of slope of Paris equation



(d) Change of initial edge crack



(e) Change in applied maximum and minimum stresses



(f) Change in slope of Paris equation

Fig. 7. Sensitivity of parameters according to the variation of random variables for the Paris and Walker models.

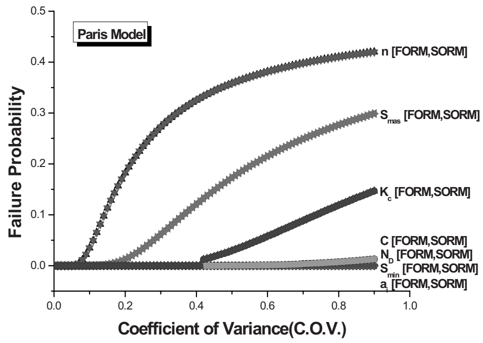
and the design fatigue life.

Although the differences among results for the variation of design fatigue life and the stress ratio are large, the differences are not distinguished clearly in Fig. 6, because the absolute values of the failure probability estimated by the FORM, the SORM and the MCS are very small.

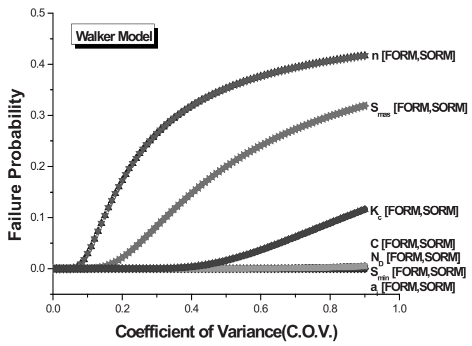
Some typical diagrams showing the effects of each random variable on the failure probability are shown in Fig. 7 using the sensitivity index. The slope of Pairs equation, n, affects significantly the failure probability with a variation of the initial edge crack size, the applied maximum and minimum stresses in the Paris and Walker models. However, the effects of the slope of Paris equation, n, on the failure probability become larger with increases of the initial edge crack size. On the other hand, the effects of other random variables such as the coefficient of Paris equation, C, the applied maximum stress, $S_{\max}$, the initial edge crack size, a_i, the applied minimum stress, $S_{\min}$, the design fatigue life, N_D and the fracture toughness, K_C, on the failure probability

Table 2. Comparison of the mean percentile differences among results obtained by using the FORM, the SORM and the MCS.

		FORM vs. MCS [%]	SORM vs. MCS [%]	FORM vs. SORM [%]
Paris Model	Initial Edge Crack	2.3182	2.3183	5.1E-04
	Maximum Stress	4.4140	4.4141	2.6E-04
	Minimum Stress	3.6939	3.6939	2.2E-05
	Design Fatigue Life	6.6352	6.6353	1.3E-04
	Slope of Paris Eq.	0.5834	0.5834	1.5E-04
Walker Model	Initial Edge Crack	4.8437	4.8437	3.7E-05
	Maximum Stress	1.0207	1.0207	1.4E-05
	Minimum Stress	4.8429	4.8429	6.9E-06
	Design Fatigue Life	1.1532	1.1532	6E-06
	Slope of Paris Eq.	1.4407	1.4407	9E-06



(a) Paris model



(b) Walker model

Fig. 8. Relationship between failure probability and C.O.V. with varying parameter for Paris and Walker models.

become smaller with increases of the initial edge crack size. Therefore, it is essential that a material constant such as the slope of the Paris equation, n, and the coefficient of Paris equation, C, must be estimated very carefully by adopting the appropriate fatigue experiment.

And it is also found from Fig. 7 that the effects of random variables on the failure probability are the same during before and after iteration with the change in the initial edge crack size, the applied maximum and minimum stresses in the Paris and Walker models. However, the design fatigue life affects significantly the failure probability before iteration with the variation of the slope of the Paris equation. The slope of the Paris equation is found to affect significantly the failure probability after iteration in the Paris and Walker models.

Fig. 8 shows the failure probability with different C.O.V. for various parameters in the Paris and Walker models. The meaning of having a larger C.O.V. is that the distribution of variables is more scattered from the mean value. It is noted in Fig. 8 that in the scattering of the slope of the Paris equation, the maximum stress and stress intensity factor affects the failure probability significantly in the Paris and Walker models. That is, the effects of experimental data scattering characteristic of the slope of the Paris equation, maximum stress and stress intensity factor on the failure probability seem to be most significant. On the other hand, it is noted in Fig. 8 that the effects of the scattering of the coefficient of Paris equation, design fatigue life, minimum stress and initial edge crack on the failure probability is not much pronounced.

Especially, it is found that the failure probability steeply increases to 4.52E18[%] with varying the stress intensity factor with the C.O.V. of approximately 0.41 for the Paris model. And we can also note that the failure probability steeply increases to 3.04E18[%] with C.O.V. of approximately 0.31 of stress intensity factor for the Walker model, although it is not shown distinctly in Fig. 8(b).

Therefore, the engineer must choose proper values of various parameters in considering the results obtained from failure probability analysis for the structure design process and the maintenance investigation. Especially, some variables need more investigation to be selected in accordance to the sensitivity analysis and C.O.V. analysis.

6. Conclusions

In this paper, the fatigue crack growth models suggested by Pairs and Walker are used to formulate the limit state function (LSF). The FORM (first order reliability method) and the SORM (second order reliability method) are used to estimate the failure probability. And the MCS (Monte Carlo simulation) is used to evaluate the applicability of the FORM and the SORM by comparing the failure probability.

Moreover, the effects of various random variables on the failure probability are systematically investigated by using the sensitivity index and the following results are obtained:

(1) The failure probability decreases with the increase of the design fatigue life and the applied minimum stress, the decrease of initial edge crack size, the applied maximum stress and the slope of Paris equation.

(2) The results out of the FORM and the SORM show the similar failure probability for the Paris and the Walker models.

(3) The slope of Pairs equation affects significantly the failure probability with the variation of random variables in the Paris and Walker models.

(4) The scattering of the data such as the slope of Paris equation, the maximum stress and the stress intensity factor in Paris and Walker models affects the failure probability significantly.

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References

[1] I. S. Stephens, F. Ali, R. S. Robert and O. F. Henry, Metal Fatigue in Engineering, John Wiley & Sons, (2001).

[2] M. S. Cheung and W. C. Li, Probabilistic fatigue and fracture analyses of steel bridges, *Structural Safety*, 23 (2003) 245-262.

[3] R. L. Steven, M. D. Grace, R. Faith, L. B. Randall, H. Amy, D. H. Scott and W. S. William, ASM Handbook, Fatigue and Fracture, ASM International, 19, (1996).

[4] T. L. Anderson, Fracture Mechanics: Fundamentals and Applications, CRC Press, (2005).

[5] T. D. Righiniotis and M. K. Chryssanthopoulos, Probabilistic fatigue analysis under constant amplitude loading, *Journal of Constructional Steel Research*, 59 (2003) 867-886.

[6] O. S. Lee, D. H. Kim and S. S. Choi, Reliability of Buried Pipeline Using A Theory of Probability of Failure, *SOLID STATE PHENOMENA*, 110 (2006) 221-230.

[7] O. S. Lee and D. H. Kim, The Reliability Estimation of Pipeline Using FORM, SORM and Monte Carlo Simulation with FAD, *Journal of Mechanical Science and Technology*, 20 (12) (2006) 2124-2135.

[8] M. Ahammed, Probabilistic estimation of remaining life of a pipeline in the presence of active corrosion defects, *International Journal of Pressure Vessels and piping*, 75 (4) (1998) 321-329.

[9] S. Mahadevan and A. Haldar, Reliability Assessment Using Stochastic Finite Element Analysis, John Wiley & Sons, (2000).

[10] S. Mahadevan and A. Haldar, Probability, Reliability and Statistical Method in Engineering Design, John Wiley & Sons, (2000).

Appendix

We have listed the typical data among the data shown in Fig. 6.

Table 3. The failure probability obtained from the FORM, the SORM and the MCS according to the initial crack for the Paris model.

Initial Crack(a_i)	FORM	SORM	MCS
0.00001	0	0	0
0.00002	0	0	0
0.00003	0	0	0
0.00004	0	0	0
0.00005	0	0	0
0.00006	0	0	0
0.00007	0	0	0
0.00008	1.57E-25	1.57E-25	0

0.00009	4.68E-22	4.68E-22	0
0.0001	2.44E-19	2.44E-19	0
0.00011	3.71E-17	3.71E-17	0
0.00012	2.28E-15	2.28E-15	0
0.00013	7.07E-14	7.07E-14	0
0.00014	1.29E-12	1.29E-12	0
0.00015	1.55E-11	1.55E-11	0
0.00016	1.33E-10	1.33E-10	0
0.00017	8.64E-10	8.64E-10	0
0.00018	4.49E-09	4.49E-09	0
0.00019	1.93E-08	1.93E-08	0
0.0002	7.08E-08	7.08E-08	2E-07
0.00021	2.27E-07	2.27E-07	4E-07
0.00022	6.46E-07	6.46E-07	6E-07
0.00023	1.67E-06	1.67E-06	1.9E-06
0.00024	3.94E-06	3.94E-06	3.9E-06
0.00025	8.61E-06	8.61E-06	9.7E-06
0.00026	1.76E-05	1.76E-05	1.51E-05
0.00027	3.4E-05	3.4E-05	3.31E-05
0.00028	6.21E-05	6.21E-05	5.99E-05
0.00029	0.000108	0.000108	0.000109
0.0003	0.000181	0.000181	0.000188
0.00031	0.000291	0.000291	0.000288
0.00032	0.000453	0.000453	0.000452
0.00033	0.000683	0.000683	0.000692
0.00034	0.001001	0.001001	0.001004
0.00035	0.00143	0.00143	0.001442
0.00036	0.001996	0.001996	0.001995
0.00037	0.002727	0.002727	0.002745
0.00038	0.003654	0.003654	0.00361
0.00039	0.004808	0.004808	0.004758
0.0004	0.006222	0.006222	0.006179
0.00041	0.00793	0.00793	0.007897
0.00042	0.009965	0.009965	0.009895
0.00043	0.012359	0.012359	0.012217
0.00044	0.015144	0.015144	0.015033
0.00045	0.018347	0.018347	0.018285
0.00046	0.021997	0.021997	0.021967
0.00047	0.026115	0.026115	0.026089
0.00048	0.030723	0.030723	0.030586
0.00049	0.035837	0.035838	0.035863
0.0005	0.041471	0.041471	0.041357
0.00051	0.047634	0.047634	0.047512
0.00052	0.054332	0.054332	0.054239
0.00053	0.061565	0.061565	0.06136
0.00054	0.069333	0.069333	0.068962
0.00055	0.077629	0.07763	0.077431
0.00056	0.086445	0.086446	0.086303
0.00057	0.095769	0.095769	0.095469
0.00058	0.105585	0.105585	0.105278
0.00059	0.115876	0.115876	0.115592
0.0006	0.126622	0.126622	0.126498
0.00061	0.1378	0.1378	0.137453
0.00062	0.149387	0.149387	0.148912
0.00063	0.161358	0.161358	0.160783
0.00064	0.173685	0.173685	0.173264
0.00065	0.186342	0.186342	0.185914
0.00066	0.199299	0.1993	0.199009
0.00067	0.21253	0.21253	0.212128
0.00068	0.226004	0.226004	0.225597
0.00069	0.239693	0.239693	0.239012
0.0007	0.253568	0.253568	0.252994
0.00071	0.267601	0.267601	0.266707
0.00072	0.281765	0.281765	0.281163
0.00073	0.296032	0.296033	0.295817
0.00074	0.310377	0.310377	0.310124
0.00075	0.324773	0.324773	0.324153
0.00076	0.339197	0.339197	0.338815
0.00077	0.353625	0.353625	0.353108
0.00078	0.368034	0.368035	0.367484
0.00079	0.382405	0.382405	0.381995
0.0008	0.396715	0.396716	0.396193

Table 4. The failure probability obtained from the FORM, the SORM and the MCS according to the initial crack for the Walker model.

Initial Crack(a_i)	FORM	SORM	MCS
0.00001	0	0	0
0.00002	0	0	0
0.00003	0	0	0
0.00004	0	0	0
0.00005	0	0	0
0.00006	0	0	0
0.00007	0	0	0
0.00008	7.1E-27	7.1E-27	0
0.00009	1.64E-23	1.64E-23	0
0.0001	7.63E-21	7.63E-21	0
0.00011	1.12E-18	1.12E-18	0
0.00012	6.96E-17	6.96E-17	0
0.00013	2.25E-15	2.25E-15	0

0.00014	4.38E-14	4.38E-14	0
0.00015	5.66E-13	5.66E-13	0
0.00016	5.27E-12	5.27E-12	0
0.00017	3.74E-11	3.74E-11	0
0.00018	2.12E-10	2.12E-10	0
0.00019	9.97E-10	9.97E-10	0
0.0002	3.99E-09	3.99E-09	0
0.00021	1.39E-08	1.39E-08	0
0.00022	4.32E-08	4.32E-08	0
0.00023	1.21E-07	1.21E-07	0
0.00024	3.09E-07	3.09E-07	0
0.00025	7.31E-07	7.31E-07	0
0.00026	1.61E-06	1.61E-06	0.000001
0.00027	3.34E-06	3.34E-06	0.000001
0.00028	6.55E-06	6.55E-06	0.000007
0.00029	1.22E-05	1.22E-05	0.000008
0.0003	2.18E-05	2.18E-05	0.000028
0.00031	3.74E-05	3.74E-05	0.000036
0.00032	6.18E-05	6.18E-05	0.00006
0.00033	9.89E-05	9.89E-05	0.000079
0.00034	0.000153	0.000153	0.000145
0.00035	0.000232	0.000232	0.000242
0.00036	0.000341	0.000341	0.000375
0.00037	0.000491	0.000491	0.000441
0.00038	0.000691	0.000691	0.000681
0.00039	0.000955	0.000955	0.000885
0.0004	0.001295	0.001295	0.001237
0.00041	0.001727	0.001727	0.001807
0.00042	0.002267	0.002267	0.002218
0.00043	0.002933	0.002933	0.00285
0.00044	0.003745	0.003745	0.003787
0.00045	0.004722	0.004722	0.004706
0.00046	0.005884	0.005884	0.005861
0.00047	0.007252	0.007252	0.007109
0.00048	0.008846	0.008846	0.008926
0.00049	0.010689	0.010689	0.010672
0.0005	0.012798	0.012798	0.012814
0.00051	0.015195	0.015195	0.014971
0.00052	0.017897	0.017897	0.017975
0.00053	0.020922	0.020922	0.020669
0.00054	0.024286	0.024286	0.024065
0.00055	0.028003	0.028003	0.02819
0.00056	0.032086	0.032086	0.032153
0.00057	0.036546	0.036546	0.03623
0.00058	0.041392	0.041392	0.040996
0.00059	0.04663	0.04663	0.046386
0.0006	0.052266	0.052266	0.052123
0.00061	0.058303	0.058303	0.058236
0.00062	0.064742	0.064742	0.06493
0.00063	0.071581	0.071581	0.071496
0.00064	0.078817	0.078818	0.078649
0.00065	0.086447	0.086447	0.086468
0.00066	0.094463	0.094463	0.094434
0.00067	0.102858	0.102858	0.102116
0.00068	0.11162	0.11162	0.111237
0.00069	0.12074	0.12074	0.1202
0.0007	0.130205	0.130205	0.13093
0.00071	0.14	0.14	0.139603
0.00072	0.150112	0.150112	0.149373
0.00073	0.160524	0.160524	0.160671
0.00074	0.171221	0.171221	0.170801
0.00075	0.182184	0.182185	0.181882
0.00076	0.193397	0.193397	0.192738
0.00077	0.204842	0.204842	0.204336
0.00078	0.216499	0.216499	0.215963
0.00079	0.228351	0.228351	0.227878
0.0008	0.240378	0.240378	0.240098
0.00081	0.252562	0.252562	0.251139
0.00082	0.264885	0.264885	0.264173
0.00083	0.277328	0.277328	0.276747
0.00084	0.289873	0.289873	0.289117
0.00085	0.302501	0.302501	0.301708
0.00086	0.315197	0.315197	0.314321
0.00087	0.327941	0.327941	0.327522
0.00088	0.340719	0.340719	0.341204
0.00089	0.353514	0.353514	0.353195
0.0009	0.36631	0.36631	0.365977
0.00091	0.379092	0.379092	0.378541
0.00092	0.391847	0.391847	0.39126
0.00093	0.40456	0.40456	0.403542
0.00094	0.417219	0.417219	0.416584
0.00095	0.429811	0.429811	0.429267
0.00096	0.442324	0.442324	0.441414
0.00097	0.454748	0.454748	0.454128
0.00098	0.467071	0.467071	0.46706
0.00099	0.479285	0.479285	0.477906
0.001	0.49138	0.49138	0.490439